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The Onset of Star Formation:

In studying stellar evolution, there are three phases that are best addressed individually. To begin with, we have to understand how stars with different masses form out of gas in the interstellar medium (ISM). We do not currently have a comprehensive theory of star formation that allows us to understand the initial epochs in the history of the star. We shall discuss this topic briefly here.

The ISM contains large molecular clouds of masses in the range $(10^5 - 10^6)\,M_\odot$, with temperatures of $(10-100)\,K$ and densities of $(10^{-24} - 10^{-22})\,g\,cm^{-3}$. To form stars by collapsing such gas clouds requires an increase of density by a factor of $\sim 10^{24}$ and an icrease in temperature by a factor of $\sim 10^6$. Here we provide a simple picture of a possible scenario.

The starting point is relevant equations that govern the dynamics of a gas (more generally, a fluid):

$$\rho \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla} P - \rho \vec{\nabla} \Phi \qquad \text{(Euler's equation)}$$

$$\vec{\nabla}\cdot(\rho \vec{v}) + \frac{\partial \rho}{\partial t} = 0 \qquad \text{(Continuity equation)}$$

$$\nabla^2 \Phi = 4\pi G \rho \qquad \text{(Poisson's equation)}$$

$$P = c_s^2 \rho \qquad \text{(Equation of state)}$$

Here ρ, P are the density and pressure of the gas, respectively, $\vec{v}$ is the velocity, Φ is the gravitational potential, and c_s is the speed of sound of the gas.

Now consider a constant background with ρ_0, P_0 and zero velocity. The gravitational potential is a constant in this case, that can be set to zero. Small perturbations around this background are defined as:

$\rho = \rho_0 + \rho_1$, $P = P_0 + P_1$, $\vec{v} = \vec{v}_1$, $\Phi = \Phi_1$

Where $\rho_1 \ll \rho_0$, $P_1 \ll P_0$. Taking the time derivative of the continuity equation and spatial derivative of the Euler's equation, we find:

$$\frac{\partial^2 \rho_1}{\partial t^2} = \nabla^2 P_1 + \rho_0 \nabla^2 \Phi_1$$

Here we have kept only the leading order terms that are linear in perturbations. After using the Poisson's equation and the equation of state, we arrive at the following equation for ρ_1:

$$\boxed{\frac{1}{c_s^2}\frac{\partial^2 \rho_1}{\partial t^2} - \nabla^2 \rho_1 = \frac{4\pi G}{c_s^2}\rho_0 \rho_1}$$

The left-hand side of this equation resembles the wave equation. Lets consider a plane-wave solution:

$$\rho_1 = \delta \, Re\left[e^{i(\vec{k}\cdot\vec{x} - \omega t)}\right]$$

We can then find the following dispersion relation:

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

It is seen that for $k^2 > \frac{4\pi G\rho_0}{c_s^2}$ we have oscillatory solutions while for $k^2 < \frac{4\pi G\rho_0}{c_s^2}$ we have exponentially growing (and damped) solutions. The borderline between the two cases is defined by Jeans wavenumber k_J:

$$k_J = \frac{\sqrt{4\pi G\rho_0}}{c_s} \Rightarrow \boxed{\lambda_J = \sqrt{\frac{\pi}{G\rho_0}}\, c_s} \; *$$

For perturbations that have a size larger than the Jeans wavelength ($\lambda > \lambda_J$) gravity wins over pressure. These perturbations will undergo gravitational collapse. On the other hand, pressure is dominant for perturbations that have size smaller than the Jeans wavelength ($\lambda < \lambda_J$). These perturbations have the familiar oscillatory behaviour known from the wave equations.

This can be intuitively seen in a number of ways:

(1) Consider a spherical cloud of mass M and radius R with uniform density ρ. The radius obeys the following equation:

$$\frac{d^2 R}{dt^2} = \frac{-GM}{R^2} - \frac{1}{\rho}\frac{dP}{dR}$$

If the second term on the right-hand side dominates, the cloud will expand due to pressure. The relevant time scale is given by (recall that $P = c_s^2 \rho$):

$$t_s \sim \frac{R}{c_s} \propto \left(\frac{M}{\rho}\right)^{\frac{1}{3}} T^{-\frac{1}{2}} \propto M^{\frac{1}{3}} \rho^{-\frac{1}{3}} T^{-\frac{1}{2}}$$

On the other hand, if the first term on the right-hand side is dominant, the cloud will collapse due to gravity. The relevant time scale for such a "free fall" is:

$$t_{ff} \sim \left(\frac{GM}{R^3}\right)^{-\frac{1}{2}} \propto (G\rho)^{-\frac{1}{2}}$$

The borderline is defined by $t_{ff} \sim t_s$, whic results in:

$$(G\rho_0)^{-\frac{1}{2}} \sim M^{\frac{1}{3}} \rho^{-\frac{1}{3}} T^{-\frac{1}{2}} \Rightarrow \frac{1}{G\rho_0} \sim \frac{R^2}{T} \Rightarrow \boxed{R \sim \sqrt{\frac{T}{G\rho_0}}} \quad **$$

For larger values of R gravity wins (hence Collapse), while for smaller values pressure wins (hence expansion). The value of R given above is the same as λ_J in equation * (up to numerical factors). To see this, consider the equation of state of an ideal gas:

$$P = \frac{\rho}{\nu m_u} k_B T \Rightarrow c_s = \sqrt{\frac{k_B T}{\nu m_u}}$$

It is seen that $R \propto \sqrt{\frac{T}{\rho_0}}$, and the same holds for λ_J once we use the above expression for c_s.

(2) Another way to see how λ_J emerges is to use Virial theorem. In equilibrium condition it states that $2K + \Omega = 0$ (K being kinetic energy of the gas, Ω being the gravitational potential energy of the cloud). For an ideal monatomic

gas we have:

$$K = \frac{3}{2} N k_B T = \frac{3}{2} \frac{M}{\nu m_u} k_B T$$

For a spherical cloud with uniform density:

$$\Omega = -\frac{3}{5} \frac{GM^2}{R}$$

Therefore:

$$2K = -\Omega \Rightarrow \frac{3M}{\nu m_u} k_B T = \frac{3}{5} G \frac{M^2}{R} \qquad (M = \frac{4}{3}\pi R^3 \rho)$$

$$\Rightarrow \boxed{R \sim \sqrt{\frac{k_B T}{\nu m_u G \rho_0}}} \quad ***$$

This is similar to equation *. For larger values of R we have $2K + \Omega < 0$, which results in contraction, while smaller values of R lead to $2K + \Omega > 0$ thus expansion.

The Jeans mass M_J is defined as the mass contained within a Jeans wavelength:

$$M_J = \frac{4}{3} \pi \lambda_J^3 \rho_0$$

Only clouds with a mass $M > M_J$ can undergo contraction, which is needed to increase their temperature and set off nuclear reactions. The Jeans mass can be cast in the following form:

$$\boxed{M_J \simeq 1.2 \times 10^{5}\ M_\odot \left(\frac{T}{100\,K}\right)^{3/2} \left(\frac{\rho_0}{10^{-24}\ g\,cm^{-3}}\right)^{-1/2} \mu^{-3/2}}$$

For typical values of T and ρ_0 in the ISM ($T \sim 100\,K$, $\rho_0 \sim 10^{-24}\ g\,cm^{-3}$), we find:

$M_J \simeq 10^5\ M_\odot$

With the corresponding Jeans wavelength of $\lambda_J \sim 10^{21}$ cm (~ 1000 lyr). This implies that only huge clouds with $M \gtrsim 10^5\ M_\odot$ will undergo contraction. A question now arises that how we can describe formation of the stars in the observed mass range $(0.1-60)\ M_\odot$ as a result of collapsing clouds.

The important thing is that ρ_0, T do not remain constant during the collapse. Obviously ρ_0 increases as a result of contraction. Behaviour of T is crucial to understand how M_J changes during the collapse.

Contraction will increase the internal energy of the gas cloud and heats it up. If the gas can radiate energy quickly, T remains constant and contraction will be isothermal. This occurs if the cooling time scale t_{cool} is shorter than the contraction time scale (which is represented by t_{ff}).

During an isothermal contraction ρ_0 increases and T is constant, hence M_J decreases.

The decrease in M_J opens up the possibility that lower mass objects, which should act as progenitors of stars, arise due to fragmentation inside the collapsing cloud.

Note, however, that M_J will not decrease indefinitely during the collapse. As the cloud becomes denser, it will start to be optically thick to radiation. Eventually, the isothermality condition will be violated. At that point contraction becomes adiabatic, for which we have:

$P \propto T^{5/2} \propto \rho_0^{5/3} \Rightarrow T \propto \rho_0^{2/3}$

Using this, we can find the dependence of the Jeans mass on ρ_0 for adiabatic contraction:

$M_J \propto \rho_0^{1/2}$

This implies that M_J increases in the adiabatic case. This allows us to determine when the process of fragmentation will cease to be effective. The smallest mass scale that

that can form because of fragmentation corresponds to M_J at the transition from isothermal to adiabatic contraction. This lower limit can be estimated as follows. The rate of radiation of energy has to be (in order to maintain isothermality):

$$A \approx \frac{E}{t_{ff}} \qquad \left(E \sim \frac{GM^2}{R}, \; t_{ff} \sim \frac{1}{\sqrt{G\rho_0}}\right)$$

$$\Rightarrow A \approx \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \frac{G^{3/2} M^{5/2}}{R^{5/2}}$$

However, a body in thermal equilibrium cannot radiate at a rate higher than that of a blackbody. The rate of radiation loss by a cloud fragment can be written as:

$$B = (4\pi R^2) f \sigma T^4 \qquad (f<1 \text{ is a correction factor})$$

For isothermal collapse $B \gg A$. Transition to adiabaticity happens when $A \approx B$. This occurs at:

$$M^5 = \left(\frac{64\pi^3}{3}\right)\left(\frac{\sigma^2 f^2 T^8 R}{G^3}\right)$$

We can now estimate M_J at the time of transition:

$$\boxed{M_J = 0.02\, M_\odot \frac{T^{\frac{1}{4}}}{f^{\frac{1}{2}}\, \mu^{\frac{9}{4}}}}$$

(T is in Kelvin)

For $T \sim 10^3$ K and $f \sim 0.1$, we get $\underline{\underline{M \sim 0.36\, M_\odot}}$. Note that the result does not change much if the parameters are varied within a reasonable range, because of mild dependence of M_J on f and T.

We therefore see that the collapse of a cloud can lead to fragments with masses of the order of the solar mass or above, but not significantly below.